If we consider two identical and indistinguishable particles, p and q at positions $\vec{r_1}$ and $\vec{r_2}$ respectively, that are in a state such that their wavefunctions overlap. This means we can represent the entire system with a single wavefunction that has a probability distribution given by $|\psi(\vec{r_1}, \vec{r_2})|^2$. If we then exchange the particles p and q so that they swap positions, the probability distribution should remain unchanged because the two particles are indistinguishable. This is shown by

$$|\psi(\vec{r_1}, \vec{r_2})|^2 = |\psi(\vec{r_2}, \vec{r_1})|^2. \tag{1}$$

Let us define an exchange operator $\hat{E}$, such that

$$\hat{E}\psi(\vec{r_1}, \vec{r_2}) = k\psi(\vec{r_2}, \vec{r_1}) \tag{2}$$

where the operator acts to exchange p and q, and k is the eigenvalue. If we operate $\hat{E}$ on the system twice, then we would expect the system to return to it's original state as given by

$$\hat{E}^2\psi(\vec{r_1}, \vec{r_2}) = k^2\psi(\vec{r_1}, \vec{r_2}) = \psi(\vec{r_1}, \vec{r_2}) \tag{3}$$

which implies that $k^2 = \pm 1$. This can be further justified by the fact that in three dimensions, a rotation of one particle around another is topologically equivalent to the identity. In other words, any rotation of one particle around another can be continuously deformed to the case where the particle doesn't move. Also, we can say that the rotation around a particle is equivalent to two particle exchanges (with a spatial translation that is unimportant), this implies that two particle exchanges is equal to 1 (the identity). Substituting k^2 back into Eq.2 we get

$$\hat{E}\psi(\vec{r_1}, \vec{r_2}) = \pm\psi(\vec{r_2}, \vec{r_1}). \tag{4}$$

We can see that this is in agreement with our condition in Eq.1 and that it is possible to have two types of particles obeying two types of symmetry. One with a symmetric (where the arbitrary phase is $e^{i0} = 1$) and the other with an antisymmetric ($e^{i\pi} = -1$) wavefunction. Those particles with symmetric wavefunctions are called the bosons, and those with antisymmetric wavefunctions are called the fermions.