1. The wave function $\Psi(\mathbf{r_1}, \mathbf{r_2})$ of two fermions must be anti-symmetric by definition of the Fermi-Dirac Statistics (See Additional Exercise 1). This gives

$$\Psi(\mathbf{r_1}, \mathbf{r_2}) = -\boldsymbol{\Psi}(\mathbf{r_2}, \mathbf{r_1})$$

when two fermions are exchanged. We can then split these multi-particle wave functions into the product of the individual fermion's (p and q) wave functions

$$\Psi_p(\mathbf{r_1})\boldsymbol{\Psi}_\mathbf{q}(\mathbf{r_2}) = -\boldsymbol{\Psi}_\mathbf{p}(\mathbf{r_2})\boldsymbol{\Psi}_\mathbf{q}(\mathbf{r_1})$$

If we then consider the case where the two fermions have the same quantum number so that $p = q$, we arrive at

$$\Psi_p(\mathbf{r_1})\boldsymbol{\Psi}_\mathbf{p}(\mathbf{r_2}) = -\boldsymbol{\Psi}_\mathbf{p}(\mathbf{r_2})\boldsymbol{\Psi}_\mathbf{p}(\mathbf{r_1})$$
$$\Rightarrow \Psi_p(\mathbf{r_1})\boldsymbol{\Psi}_\mathbf{p}(\mathbf{r_2}) = \mathbf{0}$$

which is the Pauli exclusion principle. Hence, they are compatible.

2. In one dimension it is possible to construct a non trivial topological configuration using a point enclosed by two points.

   In two dimensions it is possible to construct a non trivial topological configuration using a circle (a one-sphere) and a point.

   In three dimensions it is possible to construct a non trivial topological configuration using a spherical shell (a two-sphere) and a point.

   By extrapolation, in four dimensions it should be possible to construct a non trivial topological configuration using a solid sphere (three-sphere) and a point.

3. For the potential

$$V(\mathbf{r}) = V_{xy}(x, y) + V_z(z) + g(y.z).$$

If $g(y, z)$ is small enough that we can say $V(\mathbf{r}) \approx V_{xy}(x, y) + V_z(z)$, this allows us to use the separation of variables method to break down the wave function into

$$\Psi = \Psi_{xy}\Psi_z$$

We can then solve a 2D Schrodinger Equation for the xy-plane and treat it independently of the z-direction. What we can deduce from this is, as long as the perturbation (which can be taken to represent environmental interference in our system) is small we can effectively model anyonic systems in two dimensions. For a large perturbation, however, anyonic behaviour isn't exhibited in the system.