1. Unfortunately, the Hamiltonian given in the book is for this question is incorrect. The correct one is given by

$$H = \begin{pmatrix} \Delta_1 & \Omega_1 & \Omega_2 \\ \Omega_1^* & \Delta_2 & \Omega_3 \\ \Omega_2^* & \Omega_3^* & \Delta_3 \end{pmatrix} =$$

$$= \begin{pmatrix} sin^2(\theta_1)cos^2(\theta_2) & e^{i(\phi_1-\phi_2)}sin(\theta_1)sin(\theta_2)cos(\theta_2) & e^{i\phi_1}sin(\theta_1)cos(\theta_1)cos^2(\theta_2) \\ e^{i(\phi_2-\phi_1)}sin(\theta_1)sin(\theta_2)cos(\theta_2) & sin^2(\theta_2) & e^{i\phi_2}sin(\theta_2)cos(\theta_2)cos(\theta_1) \\ e^{-i\phi_1}sin(\theta_1)cos(\theta_1)cos^2(\theta_2) & e^{-i\phi_2}sin(\theta_2)cos(\theta_2)cos(\theta_1) & cos^2(\theta_1)cos^2(\theta_2) \end{pmatrix}$$

For further information on where the above matrix comes from and the physics involved in the question we point the reader to the additional questions offered on the books website and to sections 3 and 4 of the research paper that inspired the question: *Quantum Computation by Geometrical Means* by Jiannis Pachos, available at `https://arxiv.org/abs/quant-ph/0003150`.

2. The Lorentz force is given by $F_L = q\vec{v} \times \vec{B}$. By applying this force to an electron in motion across the plane, it is seen from the cross product that the Lorentz force acts perpendicular to the electron's direction of motion at all points in time. It is clear to see that this will result in the electron being moved on a circular path. By considering the centripetal force experienced by the electron in moving on a circular path, $F_c = \frac{m_e v^2}{r}$ with r as the radius of the circular path traversed, the frequency of the circling electron can be deduced by equating this with the Lorentz force, as below. As the magnetic field B is orthogonal to the direction of motion of the electron, $\vec{v} \times \vec{B} = |\vec{v}||\vec{B}| = vB$. So

$$\frac{m_e v^2}{r} = evB.$$

Which can be rearranged to give

$$\frac{v}{r} = \frac{eB}{m_e} = \omega$$

From this we can see that $v = r\omega$. The quantum description of Landau Levels in the book gives

$$\Psi(x,y,z) = \Psi_n^{SHO}(x-x_0)\sqrt{\frac{1}{L_2}}e^{ik_y y}\sqrt{\frac{1}{L_3}}e^{ik_z z},$$

with energy eigenvalues given by

$$E = (n+\frac{1}{2})\hbar\omega + \frac{\hbar^2}{2m_e}k_z^2$$

If we consider the energies of classical and quantum descriptions respectively they are given by

$$E_{class} = \frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2, \quad E_{quan} = (n+\frac{1}{2})\hbar\omega$$

where we have set $k_z = 0$ because we are considering the case where we are confined to two-dimensional circular motion. If we set $E_{class} = E_{quan}$ and rearrange we get

$$n = \frac{1}{2\hbar}mrv - \frac{1}{2}$$

which demonstrates what Landau level our system would need to be in before it came into agreement with classical predictions. We can see from this expression that for a radius of a classical scale (say if we set r equal to one metre), we would be in an extremely high eigenstate of the quantum description.

3. It is seen in Figure (2.11) that the electric field is orthogonal to the normal of the surface of the ribbon at all points. The dot product therefore reduces to give:

$$\oint \mathbf{E} \cdot d\mathbf{r} = \oint |E|dr = -\frac{1}{c}\frac{\partial \Phi}{\partial t}$$

This has therefore become a surface integral over the curved surface of the ribbon, which can be easily identified as

$$\oint E dr = 2\pi r T E = -\frac{1}{c}\frac{\partial \Phi}{\partial t}$$

where T is the width of the ribbon. Therefore

$$E = -\frac{1}{2\pi r T c}\frac{\partial \Phi}{\partial t}$$

Substituting from above:

$$j(t) = \frac{-\sigma_{xy}}{2\pi r T c}\frac{\partial \Phi}{\partial t}$$

Therefore it is seen that:

$$J(t) = \oint \frac{-\sigma_{xy}}{2\pi r T c}\frac{\partial \Phi}{\partial t} dr$$

This is the same surface integral as above, which is evaluated to give

$$\oint dr = 2\pi r T$$

Upon substitution this gives

$$J(t) = \frac{\sigma_{xy}}{c}\frac{\partial \Phi}{\partial t}$$

As required. Finally, we want to find the total charge Q, when the flux is increased by $\Delta\Phi = \Phi_0 = \frac{hc}{e}$. To do this, we take our expression for total current and rearrange it to give

$$\frac{J(t)c}{\sigma_{xy}} = \frac{\partial \Phi}{\partial t}$$

We the integrate both sides with respect to t to give

$$\frac{c}{\sigma_{xy}}\int J(t)dt = \int \frac{\partial \Phi}{\partial t} dt$$

$$\Rightarrow \frac{c}{\sigma_{xy}} Q = \int d\Phi$$

We can now evaluate this integral between Φ and $\Phi + \Delta\Phi$ to obtain our expression for $\Delta\Phi$

$$\frac{c}{\sigma_{xy}} Q = \int_{\Phi}^{\Phi+\Delta\Phi} d\Phi = \Phi + \Delta\Phi - \Phi = \Delta\Phi$$

Using the fact that $\Delta\Phi = \frac{hc}{e}$ we now have

$$\frac{c}{\sigma_{xy}} Q = \frac{hc}{e}$$

$$\Rightarrow Q = \frac{h}{e}\sigma_{xy}$$

as required.